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2007 J. Phys.: Condens. Matter 19 026204

(http://iopscience.iop.org/0953-8984/19/2/026204)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **19** (2007) 026204 (11pp) doi:10.1088/0953-8984/19/2/026204

Cross correlations and shot noise in a Y-shaped quantum dot

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Received 20 July 2006, in final form 10 October 2006

Published 15 December 2006

Online at stacks.iop.org/JPhysCM/19/026204

Abstract

Motivated by recent experimental realization of a Y-shaped artificial Kondo impurity, we investigated the current correlations in a three-terminal Kondo dot modelled by the Anderson Hamiltonian. Using the Keldysh nonequilibrium Green's function technique, the multiterminal noise power spectrum at zero frequency is expressed in terms of the retarded (advanced) Green's function in the dot, which is valid for small voltages and low temperatures. The retarded (advanced) Green's function is determined under the truncation beyond the Lacroix approximation by the equation of motion approach and thus describes well the nonequilibrium Kondo physics. Our numerical results, along with analytical ones in some limit cases, can be tested by present technology.

1. Introduction

Current fluctuations in mesoscopic conductors, originating from the granularity of the carriers and the thermal disturbance, have been a fruitful field of research [1]. The thermal noise does not carry extra information because it can be related to the linear conductance via the fluctuation–dissipation theorem (FDT). The shot noise (due to charge discreteness) is a purely nonequilibrium property and it can provide information additional to the averaged current. To study correlations in mesoscopic transport, shot noise is an important tool [2]. For uncorrelated carriers, shot noise is Poissonian. The Pauli exclusion principle always suppresses shot noise from the Poisson value. However, when Coulomb interaction is taken into account, shot noise can be sub-Poissonian or super-Poissonian depending on the details of the systems under consideration [3–5].

Theoretical and experimental studies on shot noise in strongly correlated electron systems are scarce, especially when the systems show a Kondo effect [6, 7]. One of the main paradigms of such systems in mesoscopic physics is the quantum-dot device. Indeed, the electronic transport through a quantum dot (QD) is highly correlated due to the Kondo effect. QDs offer new control for studying a continuous range of physically interesting situations because of the electrical tunability of their parameters. The ability to control the dot–lead couplings and the energy levels in the dot by gate voltages allows one to make an investigation of wide scope from Coulomb-blockade oscillations to the Kondo resonance, from open to closed dots. Here we only mention Meir and Golub's [6] study of the shot noise in a two-terminal QD in the Kondo regime. The authors find that the voltage-scaled shot noise exhibits a nonmonotonic dependence on voltage with a peak around the Kondo temperature, which is obtained by using five complementary approaches and should be reliable.

Recently, Leturcq *et al* [8] performed a first direct realization of a quantum ring connected to three normal leads. They managed to operate the ring, where effectively a single spin is localized, in the Kondo regime. This experiment motivated us to investigate the current cross correlations in a multiterminal Kondo impurity. To our knowledge, up to now there has been no measurement of cross correlations in a system where the Kondo effect dominates. Theoretically, the only relevant results are due to Sánchez and López [9] based on the slave-boson mean-field (SBMF) theory. In this paper, we establish closed formulae for the multiterminal noise power using the nonequilibrium Green's function (GF) technique together with the equation of motion (EOM) approach. The power spectrum is then explicitly related to the Kondo-enhanced density of states (DOS) which is calculated beyond the usual Lacroix approximation.

2. Formulation

We consider the transport through a Y-shaped QD using the Anderson impurity model, which reads

$$H = \sum_{\sigma} \varepsilon_{d\sigma} d_{\sigma}^{\dagger} d_{\sigma} + \frac{U}{2} \sum_{\sigma} \hat{n}_{d\sigma} \hat{n}_{d\bar{\sigma}} + \sum_{k,\sigma,\alpha} \varepsilon_{k\sigma\alpha} C_{k\sigma\alpha}^{\dagger} C_{k\sigma\alpha} + \sum_{k,\sigma,\alpha} \left(V_{k\sigma\alpha} C_{k\sigma\alpha}^{\dagger} d_{\sigma} + V_{k\sigma\alpha}^{*} d_{\sigma}^{\dagger} C_{k\sigma\alpha} \right), \tag{1}$$

where the first two terms describe the interacting QD with the onsite Coulomb repulsion U, the last term is the tunnelling Hamiltonian, and the third term models the three leads ($\alpha = 1, 2, 3$), each with a distribution function $f_{\alpha}(\varepsilon) = 1/[1 + \exp \beta(\varepsilon - V_{\alpha})]$ due to their different applied voltages V_{α}.

The current from the α lead to the QD is given by the time evolution of the occupation number in the α lead

$$\hat{I}_{\alpha} = -e \frac{\partial \hat{N}_{\alpha}(t)}{\partial t} = -\frac{e}{\mathrm{i}\hbar} \sum_{k,\sigma} \left[V_{k\sigma\alpha} C_{k\sigma\alpha}^{\dagger}(t) d_{\sigma}(t) - V_{k\sigma\alpha}^{*} d_{\sigma}^{\dagger}(t) C_{k\sigma\alpha}(t) \right]. \tag{2}$$

Depending on the energy properties of the noise detector, both symmetrized and nonsymmetrized definitions of the noise power spectrum are used in the literature [3, 10]. In a three-terminal device, the symmetrized shot noise and the nonsymmetrized shot noise are equivalent at zero frequency, but the corresponding cross correlations are not equivalent. Here we focus on the zero-frequency noise spectrum and adopt the symmetrized version. We thus define the correlation function of the current in the α lead and the current in the β lead as

$$S_{\alpha\beta}(t) \equiv \langle [\Delta \hat{I}_{\alpha}(t), \Delta \hat{I}_{\beta}(0)]_{+} \rangle = \langle [\hat{I}_{\alpha}(t), \hat{I}_{\beta}(0)]_{+} \rangle - 2\langle \hat{I}_{\alpha} \rangle \langle \hat{I}_{\beta} \rangle. \tag{3}$$

Inserting equation (2) into (3) one gets the following result for the current correlation function after some calculations:

$$S_{\alpha\beta}(t) = e^{2} \sum_{k,\sigma,k',\sigma'} V_{k\sigma\alpha}^{*} V_{k'\sigma'\beta} \left[G_{k\sigma\alpha,k'\sigma'\beta}^{>}(t) G_{\sigma',\sigma}^{<}(-t) + G_{k\sigma\alpha,k'\sigma'\beta}^{<}(t) G_{\sigma',\sigma}^{>}(-t) \right] + e^{2} \sum_{k,\sigma,k',\sigma'} V_{k\sigma\alpha} V_{k'\sigma'\beta}^{*} \left[G_{\sigma,\sigma'}^{>}(t) G_{k'\sigma'\beta,k\sigma\alpha}^{<}(-t) + G_{\sigma,\sigma'}^{<}(t) G_{k'\sigma'\beta,k\sigma\alpha}^{>}(-t) \right]$$

$$- e^2 \sum_{k,\sigma,k',\sigma'} V_{k\sigma\alpha} V_{k'\sigma'\beta} \left[G^{>}_{\sigma,k'\sigma'\beta}(t) G^{<}_{\sigma',k\sigma\alpha}(-t) + G^{<}_{\sigma,k'\sigma'\beta}(t) G^{>}_{\sigma',k\sigma\alpha}(-t) \right]$$
$$- e^2 \sum_{k,\sigma,k',\sigma'} V^*_{k\sigma\alpha} V^*_{k'\sigma'\beta} \left[G^{>}_{k\sigma\alpha,\sigma'}(t) G^{<}_{k'\sigma'\beta,\sigma}(-t) + G^{<}_{k\sigma\alpha,\sigma'}(t) G^{>}_{k'\sigma'\beta,\sigma}(-t) \right], \tag{4}$$

where $G^{<(>)}$ is the lesser (greater) Green's function of the Keldysh type [11] corresponding to the states at the dot and in the leads. In deriving the above equation, we have approximately expanded the four-operator correlation functions, e.g.

$$\left\langle C^\dagger_{k\sigma\alpha}(t) d_\sigma(t) C^\dagger_{k'\sigma'\beta}(0) d_{\sigma'}(0) \right\rangle \approx \left\langle C^\dagger_{k\sigma\alpha}(t) d_\sigma(t) \right\rangle \left\langle C^\dagger_{k'\sigma'\beta}(0) d_{\sigma'}(0) \right\rangle$$
$$+ \left\langle C^\dagger_{k\sigma\alpha}(t) d_{\sigma'}(0) \right\rangle \left\langle d_\sigma(t) C^\dagger_{k'\sigma'\beta}(0) \right\rangle. \tag{5}$$

Next, we use the equation of motion approach and Langreth's rules for analytic continuation [11] to express $S_{\alpha\beta}(t)$ only by the GFs G_σ at the dot and the bare GFs $g_{k\sigma\alpha}$ in the leads. The lesser, greater, retarded, and advanced GFs $g_{k\sigma\alpha}$ take the form of free fermions. After applying a Fourier transform, the noise power spectrum in the zero-frequency limit is finally obtained as

$$S_{\alpha\beta} \equiv \lim_{\omega\to 0} S_{\alpha\beta}(\omega) = \frac{8e^2}{h} \sum_\sigma \Gamma_{\sigma\alpha}\Gamma_{\sigma\beta} \int \mathrm{d}\varepsilon \big\{ G^<_\sigma(\varepsilon) G^>_\sigma(\varepsilon) + G^<_\sigma(\varepsilon) G^a_\sigma(\varepsilon) - G^<_\sigma(\varepsilon) G^r_\sigma(\varepsilon)$$
$$+ f_\alpha(\varepsilon) G^<_\sigma(\varepsilon) G^r_\sigma(\varepsilon) - f_\alpha(\varepsilon) G^>_\sigma(\varepsilon) G^a_\sigma(\varepsilon) - f_\alpha(\varepsilon) G^a_\sigma(\varepsilon) G^a_\sigma(\varepsilon)$$
$$+ f_\beta(\varepsilon) G^>_\sigma(\varepsilon) G^r_\sigma(\varepsilon) - f_\beta(\varepsilon) G^<_\sigma(\varepsilon) G^a_\sigma(\varepsilon) - f_\beta(\varepsilon) G^r_\sigma(\varepsilon) G^r_\sigma(\varepsilon)$$
$$+ f_\alpha(\varepsilon) f_\beta(\varepsilon) G^a_\sigma(\varepsilon) G^a_\sigma(\varepsilon) + f_\alpha(\varepsilon) f_\beta(\varepsilon) G^r_\sigma(\varepsilon) G^r_\sigma(\varepsilon)$$
$$- \frac{\mathrm{i}}{2\Gamma_{\sigma\alpha}} \delta_{\alpha\beta} \left[G^<_\sigma(\varepsilon) - f_\alpha(\varepsilon) G^<_\sigma(\varepsilon) - f_\alpha(\varepsilon) G^>_\sigma(\varepsilon) \right] \big\}, \tag{6}$$

where $\Gamma_{\sigma\alpha}$, defined as $\Gamma_{\sigma\alpha} \equiv \pi\rho_{\sigma\alpha}|V_{k\sigma\alpha}|^2$, is the hybridization width of the virtual bound state. One can neglect the energy dependence of $\Gamma_{\sigma\alpha}$ in the wide band limit. The density of states $\rho_{\sigma\alpha}$ for conduction electrons in the α lead is taken to be constant when $-D < \varepsilon < D$; D is the half band width. This equation is our basic formula for the zero-frequency noise power through a Y-shaped dot. It describes the thermal noise (at equilibrium), cross correlations ($\alpha \neq \beta$) and shot noise ($\alpha = \beta$). The decoupling scheme of equation (5) is indispensable to the derivation of equation (6). Equation (5) is a good approximation based on the following two considerations: (i) the d-electrons and C-electrons are two different types of electron, having different spatial configurations and less overlap of their wavefunctions, so that their dynamical correlations should be small, (ii) as U is very large, the most possible population of the d-level is that with only one electron because of the strong effective Coulomb repulsion, which in turn reduces the two-body correlations between d-electrons and C-electrons. A more sophisticated treatment beyond this approximation is still needed if one wants to take into account the effect of finite frequencies and properties far from equilibrium [1].

The four GFs, appearing in equation (6), are not independent: they obey $G^r - G^a = G^> - G^<$. However, the lesser and greater ones cannot be directly obtained by their EOMs without introducing additional assumptions. We assume that their self-energies have the form

$$\Sigma^>_\sigma(\varepsilon) = -2\mathrm{i} \sum_\alpha \Gamma_{\sigma\alpha} \left[1 - f_\alpha(\varepsilon)\right] R_\sigma(\varepsilon),$$
$$\Sigma^<_\sigma(\varepsilon) = 2\mathrm{i} \sum_\alpha \Gamma_{\sigma\alpha} f_\alpha(\varepsilon) R_\sigma(\varepsilon), \tag{7}$$

where $R_\sigma(\varepsilon)$ is the renormalization factor due to the Coulomb repulsion U in the dot. With the help of the Dyson equation for $G^{r(a)}$ and the Keldysh equation [11] for $G^{>(<)}$, one can readily relate $R_\sigma(\varepsilon)$ to the retarded self-energy: $R_\sigma(\varepsilon) = -\mathrm{Im}\,\Sigma^r_\sigma(\varepsilon)/\Gamma_\sigma$, where $\Gamma_\sigma \equiv \sum_\alpha \Gamma_{\sigma\alpha}$ is the total hybridization width. Obviously, $R_\sigma(\varepsilon) = 1$ for a noninteracting dot. Equation (7) has the advantages that (i) it is exact in the limit $U \to 0$, (ii) it allows us to write $G^{>(<)}$ in a 'pseudoequilibrium' form if we introduce a nonequilibrium distribution function $f_{\sigma 0}(\varepsilon) \equiv \sum_\alpha \Gamma_{\sigma\alpha} f_\alpha(\varepsilon)/\Gamma_\sigma$,

$$\begin{aligned} G^<_\sigma(\varepsilon) &= -f_{\sigma 0}(\varepsilon)\left[G^r_\sigma(\varepsilon) - G^a_\sigma(\varepsilon)\right], \\ G^>_\sigma(\varepsilon) &= \left[1 - f_{\sigma 0}(\varepsilon)\right]\left[G^r_\sigma(\varepsilon) - G^a_\sigma(\varepsilon)\right]. \end{aligned} \tag{8}$$

Substituting equation (8) in (6), a more compact expression for the power spectrum reads, after a rearrangement,

$$\begin{aligned} S_{\alpha\beta} = \frac{8e^2}{h} \sum_\sigma \Gamma_{\sigma\alpha}\Gamma_{\sigma\beta} \int \mathrm{d}\varepsilon \Bigg\{ &(f_{\sigma 0} - f_\alpha)\left(f_{\sigma 0} - f_\beta\right)[G^r_\sigma(\varepsilon) - G^a_\sigma(\varepsilon)]^2 \\ &+ \mathrm{i}[G^r_\sigma(\varepsilon) - G^a_\sigma(\varepsilon)]\Bigg[\frac{1}{2\Gamma_\sigma R_\sigma(\varepsilon)}\left[f_\alpha\left(f_\beta - 1\right) + f_\beta\left(f_\alpha - 1\right)\right] \\ &- \frac{\delta_{\alpha\beta}}{2\Gamma_{\sigma\alpha}}\left[f_\alpha\left(f_{\sigma 0} - 1\right) + f_{\sigma 0}\left(f_\alpha - 1\right)\right]\Bigg]\Bigg\}. \end{aligned} \tag{9}$$

In a similar way, the expectation value of equation (2), i.e. the average current, can be cast in terms of the retarded and advanced GFs,

$$I_\alpha = \frac{2e}{\mathrm{i}h} \sum_\sigma \Gamma_{\sigma\alpha} \int \mathrm{d}\varepsilon [f_{\sigma 0}(\varepsilon) - f_\alpha(\varepsilon)][G^r_\sigma(\varepsilon) - G^a_\sigma(\varepsilon)]. \tag{10}$$

Equation (10) is exact and has already taken into account the current conservation.

The implication of the current conservation in the current cross correlations is that the cross correlations are negative (positive) due to the Fermi (Bose) statistics of the carriers [1, 2]. Positive correlations can be found when the transport is spin dependent or the leads are superconductors [12–14]. In the present work, we confine ourself to the spin-independent transport, i.e. we take $G^r_\sigma(\varepsilon) = G^r_{\overline{\sigma}}(\varepsilon) = G^r(\varepsilon)$, $\Gamma_{\sigma\alpha} = \Gamma_{\overline{\sigma}\alpha} = \Gamma_\alpha$, etc. In this case, equation (9) describes the negative cross correlations well.

To determine the retarded (advanced) GF in the Kondo regime, we prefer the EOM approach among a few known methods which can be adopted for the nonequilibrium Kondo problem. The EOM approach takes into account all relevant scattering processes and even describes the Kondo effect in a quantitative way depending on a proper truncation approximation [15]. The EOM generates higher-order GFs; a systematic decoupling procedure of this infinite hierarchy of the high-order GFs is provided by the correlation dynamics [15–17]. The main idea is to employ cluster expansions, which express the high-order GFs in terms of lower-order GFs and same-order correlated GFs, such as (in the Zubarev notation)

$$\langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d^\dagger_\sigma \rangle\rangle = n_{d\overline{\sigma}} \langle\langle d_\sigma | d^\dagger_\sigma \rangle\rangle + \langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}}, \tag{11}$$

$$\langle\langle d^\dagger_{\overline{\sigma}} C_{k'\overline{\sigma}\alpha'} C_{k\sigma\alpha} | d^\dagger_\sigma \rangle\rangle = \langle d^\dagger_{\overline{\sigma}} C_{k'\overline{\sigma}\alpha'} \rangle \langle\langle C_{k\sigma\alpha} | d^\dagger_\sigma \rangle\rangle + \langle\langle d^\dagger_{\overline{\sigma}} C_{k'\overline{\sigma}\alpha'} C_{k\sigma\alpha} | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}}, \tag{12}$$

$$\begin{aligned} \langle\langle \hat{n}_{d\overline{\sigma}} C^\dagger_{k'\overline{\sigma}\alpha'} C_{k\overline{\sigma}\alpha} d_\sigma | d^\dagger_\sigma \rangle\rangle &= n_{d\overline{\sigma}} \langle\langle C^\dagger_{k'\overline{\sigma}\alpha'} C_{k\overline{\sigma}\alpha} d_\sigma | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}} + \langle C^\dagger_{k'\overline{\sigma}\alpha'} C_{k\overline{\sigma}\alpha} \rangle \langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}} \\ &+ \langle \hat{n}_{d\overline{\sigma}} C^\dagger_{k'\overline{\sigma}\alpha'} C_{k\overline{\sigma}\alpha} \rangle \langle\langle d_\sigma | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}} + \langle\langle \hat{n}_{d\overline{\sigma}} C^\dagger_{k'\overline{\sigma}\alpha'} C_{k\overline{\sigma}\alpha} d_\sigma | d^\dagger_\sigma \rangle\rangle_{\mathrm{c}}. \end{aligned} \tag{13}$$

The correlated GFs $\langle\langle \cdots \rangle\rangle_{\mathrm{c}}$ are defined such that they cannot be reduced to lower-order ones by any way of decoupling. One can find the details of the cluster expansion and the correlation dynamics in [17]. After writing down the hierarchy of EOM of the usual GFs $\langle\langle \cdots \rangle\rangle$, the above systematic cluster expansions are employed. The hierarchy of EOMs of the usual GFs is then transformed into that of EOM of the correlated GFs $\langle\langle \cdots \rangle\rangle_{\mathrm{c}}$. To save space we only present here the first two EOMs of the correlated GFs:

$$\left(\hbar\omega - \varepsilon_{d\sigma} - U n_{d\overline{\sigma}} - \sum_{k,\alpha} \frac{|V_{k\sigma\alpha}|^2}{\hbar\omega - \varepsilon_{k\sigma\alpha}}\right) \langle\langle d_\sigma | d_\sigma^\dagger \rangle\rangle = 1 + U \langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \tag{14}$$

$$\begin{aligned}[\hbar\omega - \varepsilon_{d\sigma} + U(n_{d\overline{\sigma}} - 1)] \langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} &= U(1 - n_{d\overline{\sigma}}) n_{d\overline{\sigma}} \langle\langle d_\sigma | d_\sigma^\dagger \rangle\rangle \\ &+ \sum_{k,\alpha} V_{k\sigma\alpha}^* \langle\langle \hat{n}_{d\overline{\sigma}} C_{k\sigma\alpha} | d_\sigma^\dagger \rangle\rangle_{\rm c} + \sum_{k,\alpha} V_{k\overline{\sigma}\alpha}^* \langle\langle d_{\overline{\sigma}}^\dagger C_{k\overline{\sigma}\alpha} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \\ &- \sum_{k,\alpha} V_{k\overline{\sigma}\alpha} \langle\langle C_{k\overline{\sigma}\alpha}^\dagger d_{\overline{\sigma}} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c}. \end{aligned} \tag{15}$$

The EOMs of other higher-order correlated GFs can be easily obtained by using the cluster expansion and the EOMs of the usual GFs. This hierarchy of correlated GFs itself provides a uniform and physical reasonable truncation scheme, i.e. truncation with respect to the order of correlations. The mean-field theory is reached if one assumes $\langle\langle \hat{n}_{d\overline{\sigma}} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$. Subsequently, $\langle\langle \hat{n}_{d\overline{\sigma}} C_{k\sigma\alpha} | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$, $\langle\langle d_{\overline{\sigma}}^\dagger C_{k\overline{\sigma}\alpha} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$, and $\langle\langle C_{k\overline{\sigma}\alpha}^\dagger d_{\overline{\sigma}} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$ leads to the Hubbard-I approximation. The next higher-order truncation is exactly the Lacroix approximation which neglects the correlated GFs involving two conduction-electron operators, namely, $\langle\langle d_{\overline{\sigma}}^\dagger C_{k\overline{\sigma}\alpha} C_{k'\sigma\alpha'} | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$, $\langle\langle C_{k\overline{\sigma}\alpha}^\dagger d_{\overline{\sigma}} C_{k'\sigma\alpha'} | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$, and $\langle\langle C_{k\overline{\sigma}\alpha}^\dagger C_{k'\overline{\sigma}\alpha'} d_\sigma | d_\sigma^\dagger \rangle\rangle_{\rm c} \approx 0$. Although it is essentially equivalent to the usual Tyablikov decoupling adopted by Lacroix [18, 19], our decoupling scheme, i.e. the correlation truncation scheme, is more systematic and is applicable for going beyond the Lacroix approximation. To achieve this, we take into account the three correlated GFs neglected by Lacroix and assume higher-order correlated GFs, such as the last term in the right-hand side of equation (13), to be zero. A closed set of EOMs for the correlated GFs is then obtained. After a lengthy but straightforward calculation, in the strong correlation limit $U \to \infty$, we obtain the d-electron GF beyond the Lacroix approximation as (the spin indices are omitted)

$$\begin{aligned} G^r(\varepsilon) &= \langle\langle d_\sigma | d_\sigma^\dagger \rangle\rangle \\ &= \frac{1 - n_{\rm d} - A(\varepsilon) - \frac{A^2(\varepsilon)}{n_{\rm d}}}{\varepsilon - \varepsilon_{\rm d} + {\rm i}\Gamma\left[1 - \frac{A^2(\varepsilon)}{n_{\rm d}}\right] + 2[B(\varepsilon) - {\rm i}\Gamma A(\varepsilon)] + \frac{A(\varepsilon)}{n_{\rm d}}[B(\varepsilon) - {\rm i}\Gamma A(\varepsilon)]}, \end{aligned} \tag{16}$$

with

$$A(\varepsilon) = -\frac{\Gamma}{\pi} \int_{-D}^{D} {\rm d}\varepsilon' \frac{f_0(\varepsilon') \left[G^r(\varepsilon')\right]^*}{\varepsilon' - \varepsilon - {\rm i}\eta}, \tag{17}$$

$$B(\varepsilon) = \frac{\Gamma}{\pi} \int_{-D}^{D} {\rm d}\varepsilon' \frac{f_0(\varepsilon')}{\varepsilon' - \varepsilon - {\rm i}\eta}, \tag{18}$$

$$n_{\rm d} = -\frac{\rm i}{2\pi} \int_{-D}^{D} {\rm d}\varepsilon\, G^<(\varepsilon). \tag{19}$$

In deriving equation (16), there emerge four two-body correlation functions, $\langle d_\sigma^\dagger d_{\overline{\sigma}}^\dagger C_{k'\overline{\sigma}\alpha'} C_{k\sigma\alpha} \rangle$, $\langle \hat{n}_{d\sigma} d_{\overline{\sigma}}^\dagger C_{k\sigma\alpha} \rangle$, $\langle \hat{n}_{d\overline{\sigma}} C_{k'\overline{\sigma}\alpha'}^\dagger C_{k\overline{\sigma}\alpha} \rangle$, and $\langle d_\sigma^\dagger C_{k'\overline{\sigma}\alpha'}^\dagger d_{\overline{\sigma}} C_{k\sigma\alpha} \rangle$. The first two are zero due to the fact that double occupation is forbidden in the limit of $U \to 0$. $\langle \hat{n}_{d\overline{\sigma}} C_{k'\overline{\sigma}\alpha'}^\dagger C_{k\overline{\sigma}\alpha} \rangle$ is related to $\langle \hat{n}_{d\sigma} d_{\overline{\sigma}}^\dagger C_{k\sigma\alpha} \rangle$ by the EOM and the spectral theorem and is zero subsequently. The last one can be expanded as $\langle d_\sigma^\dagger C_{k'\overline{\sigma}\alpha'}^\dagger d_{\overline{\sigma}} C_{k\sigma\alpha} \rangle = \langle d_\sigma^\dagger C_{k\sigma\alpha} \rangle \langle C_{k'\overline{\sigma}\alpha'}^\dagger d_{\overline{\sigma}} \rangle + \langle d_\sigma^\dagger C_{k'\overline{\sigma}\alpha'}^\dagger d_{\overline{\sigma}} C_{k\sigma\alpha} \rangle_{\rm c}$. The spin-flip correlation $\langle d_\sigma^\dagger C_{k'\overline{\sigma}\alpha'}^\dagger d_{\overline{\sigma}} C_{k\sigma\alpha} \rangle_{\rm c}$ is assumed to be zero for simplicity.

Notably, $G^r(\varepsilon)$ has a nontrivial dependence on the voltages through the nonequilibrium distribution function, which captures the nonequilibrium Kondo physics better than the Lacroix approximation. At equilibrium and zero temperature, $A(\varepsilon)$ and $B(\varepsilon)$ have singularities at the Fermi level, but $G^r(\varepsilon)$ varies more smoothly around this point. Equation (16) can be written as

$$G^r(E_F) = \frac{\Gamma}{\pi} \frac{[G^r(E_F)]^*}{(2i\Gamma^2/\pi)[G^r(E_F)]^* + \Gamma/\pi}, \tag{20}$$

where $E_F = 0$ is the Fermi level. Equation (20) has already been obtained by Lacroix [18, 20]. The solution of this equation is $G^r(E_F) = (\sin\theta\cos\theta)/\Gamma - i(\sin^2\theta)/\Gamma$, with an arbitrary phase θ. Since the solution has the same form of the exact Fermi-liquid relation which obeys the Friedel sum rule [21, 22], a natural and physical choice is to relate the phase to the occupation number, i.e. $\theta = \pi n_d$, as the rule reveals. Using the Dyson equation of $G^r(\varepsilon)$, equation (19) when integrated over ε from $-\infty$ to E_F gives [20, 21]

$$\theta = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\frac{\varepsilon_d}{\Gamma}. \tag{21}$$

3. Results and discussions

Our computational scheme is as follows. For a given set of parameters (ε_d, D, Γ_α) of the device, we first solve the self-consistent equations (16)–(19) at equilibrium. The self-consistently calculated occupation number n_d determines when the iteration stops. The regularization $i\eta$, introduced to cancel the singularities of $A(\varepsilon)$ and $B(\varepsilon)$ at E_F, is determined such that the final $\mathrm{Im}[G^r(E_F)]$ is in agreement with that obtained by equation (21). With this fixed η, we then solve equations (16)–(19) self-consistently in nonequilibrium by applying voltages to the leads.

Before we present numerical results on the current correlations, some general discussions are necessary. For a strongly interacting Y-shaped dot in the Kondo limit ($\varepsilon_d \ll -\Gamma$), it is known that $G^r(\varepsilon)$ consists of approximately two well-separated Lorentzian poles [23] in equilibrium. One of them is located at ε_d, and the other one is the Kondo resonance at the Fermi level E_F. The broadening effect of the impurity level on the DOS around E_F can be neglected and the renormalization factor is readily find as $R(\varepsilon) = 1$ in this case. When the dot is driven out of equilibrium by applying biases to the three leads, the broadening effect is still weak; however, $R(\varepsilon)$ deviates from 1 due to the splitting of the Kondo peak. On the other hand, if $|\varepsilon_d|$ decreases to be only several times bigger than Γ (of course, still in the Kondo regime), the broadening effect will become important. The DOS is no longer a Lorentzian around E_F, neither can it be written as a sum of Lorentzian functions [23] due to the interference between the broadening impurity level and the Kondo resonance. Equation (16) can describe these effects well. In this regime, $R(\varepsilon) = 1$ holds only around E_F even in equilibrium. Figure 1 shows $R(\varepsilon)$ in the Kondo limit (i.e. ε_d is deep below the Fermi level $\varepsilon_d \ll -\Gamma$) and in the Kondo regime with $\varepsilon_d = -5\Gamma$ for different biases. Thus, equations (9) and (10) exactly recover the FDT for the thermal (equilibrium) noise at low temperatures $S_{\alpha\beta}|_{eq} = 4k_B T\mathcal{G}_{\alpha\beta}$, where T is the temperature and $\mathcal{G}_{\alpha\beta}$ the linear conductance defined as $\mathcal{G}_{\alpha\beta} \equiv e\,\partial I_\alpha/\partial V_\beta\big|_{V_\beta=0}$. This allows us to have the confidence in the reliability of equation (9) to describe the zero-temperature correlations. Note that the equilibrium Kondo temperature T_K is defined as the half-width of the Kondo resonance in the DOS. T_K is obviously dependent on the bare level ε_d. However, for the sake of presentation, we use the same notation T_K to denote different Kondo temperatures in the Kondo regime with $\varepsilon_d = -5\Gamma$ and in the Kondo limit, as seen in figure 1.

What follows are the numerical results on the shot noise and the cross correlations at zero temperature. The leads 2 and 3 are always symmetrically biased, i.e. $V_2 = -V_3 = V/2$. For simplicity, we also assume equal tunnel couplings ($\Gamma_\alpha = \Gamma/3$). In figure 2 we plot the $S_{11} \sim V_1$ characteristic for different voltages applied to leads 2 and 3. It indicates that S_{11} at $V_1 = 0$ is nonzero with increasing V, implying a divergence of the Fano factor $\gamma_{11} \equiv S_{11}/(2e|I_1|)$. The

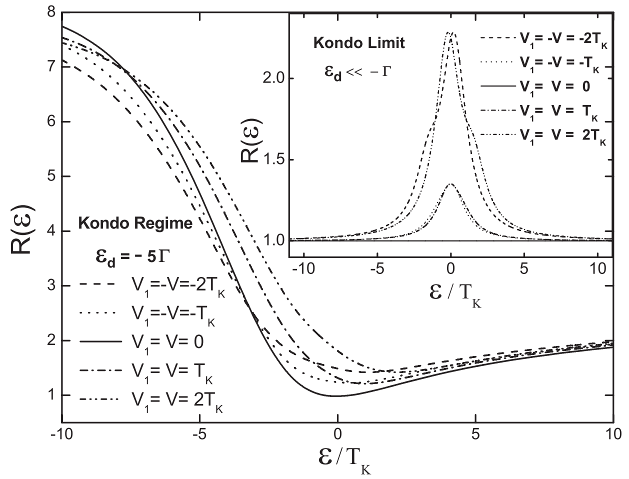

Figure 1. $R(\varepsilon)$ for different biases. The leads 2 and 3 are symmetrically biased i.e. $V_2 = -V_3 = V/2$. The renormalization factor is significantly changed by the broadening effect.

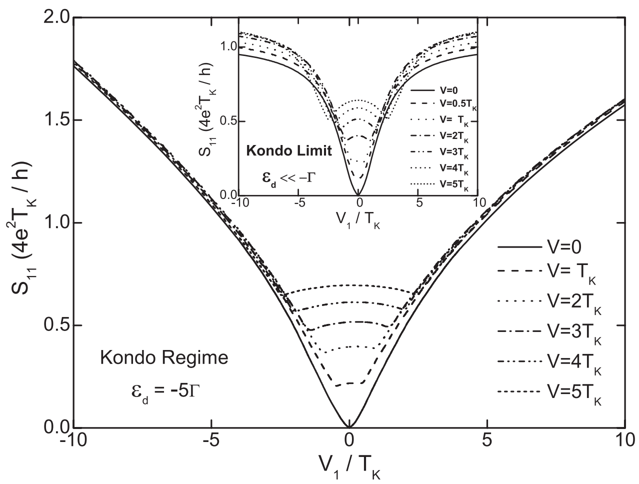

Figure 2. The shot noise in lead 1 versus bias voltage V_1 for different values of the voltage drop V.

fact that the minimum value of S_{11} at $V_1 = 0$ turns into a maximum is directly related to the Kondo physics. Namely, the maximum of the DOS at $\varepsilon = 0$ (proportional to the transmission coefficient) turns into a minimum due to the voltage-induced splitting of the Kondo peak. This feature is similar to the corresponding results of [9] obtained by the SBMF approach with the parameter $\varepsilon_d = -6\Gamma$. We also observe that the broadening effect due to the finite value of $|\varepsilon_d|/\Gamma$ on the shot noise S_{11} is significant only for large V_1. In this voltage regime, the lineshapes of figure 2 are different from the results of [9], where S_{11} drops when V_1 becomes very large as compared to T_K and thus indicates the vanishing of the Kondo effect at large voltages. By contrast, our results imply that the Kondo effect remains at weak coupling when V_1 is large [24]. Nevertheless, we cannot take results for $V_1 \gg T_K$ with great confidence, as mentioned in section 2.

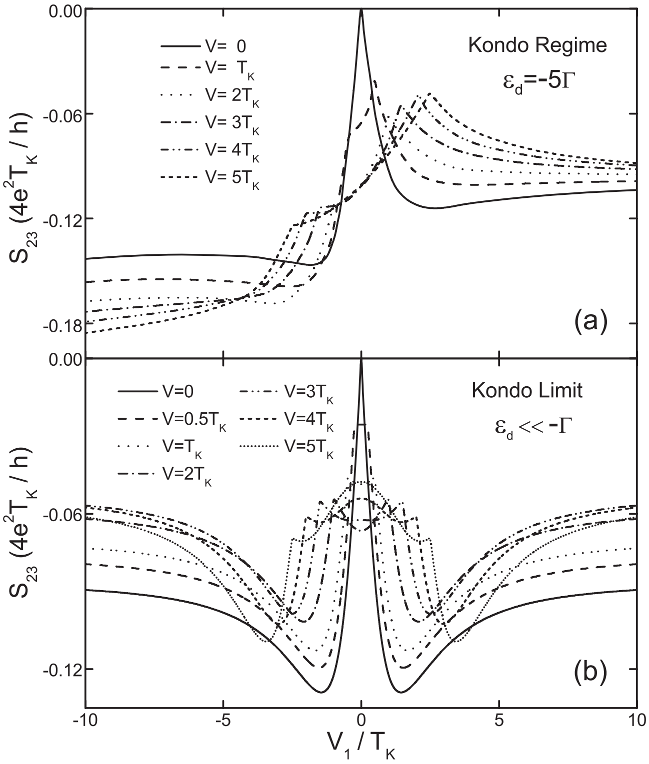

Figure 3. The cross correlations between leads 2 and 3 versus bias voltage V_1 for different values of the voltage drop V.

The feature of the cross correlations between leads 2 and 3 is more interesting, as shown in figure 3. In the Kondo limit, figure 3(b) shows that S_{23} at $V_1 = 0$ first decreases and then increases back with increasing V. The DOS is symmetric around $\varepsilon = 0$, leading to a zero net current through lead 1 at $V_1 = 0$. S_{23} depends then on the transmission coefficient T_{23} through the form $T_{23}(1 - T_{23})$ [6, 25] because lead 1 at $V_1 = 0$ acts as a voltage probe with zero impedance. At $V = 0$, the system is in the unitary limit and linear response regime, $T_{23} = 1$, and S_{23} vanishes. When V is much larger than the Kondo temperature, T_{23} is very small, and S_{23} vanishes again. Thus we have explained the nonmonotonic dependence of the cross correlations around $V_1 = 0$ on the voltage between leads 2 and 3 in the Kondo limit. Due to the broadening effect, figure 3(a) shows that S_{23} in the Kondo regime with $\varepsilon_d = -5\Gamma$ is quite different from that in the Kondo limit in the whole range of V_1. In this regime, $I_1 \neq 0$ at $V_1 = 0$, resulting from the asymmetric DOS. Consequently, the dependence of S_{23} on V_1 is rather asymmetric, and the rise of S_{23} around $V_1 = 0$ with increasing V is also hindered. From the above observations, it is clear that S_{23} is more sensitive than S_{11} to the broadening effect of the impurity level. Recent studies have revealed the important contribution of this broadening effect to the Fano resonance around the Fermi level due to the interference between the Kondo resonance and the broadening impurity level [23]. Therefore, the three-terminal QD not only provides a direct experimental access [8] to the nonequilibrium Kondo DOS but also, through the cross-correlation measurements, yields additional information on electronic correlations in the DOS, which is important for the microscopic picture for the Fano resonance.

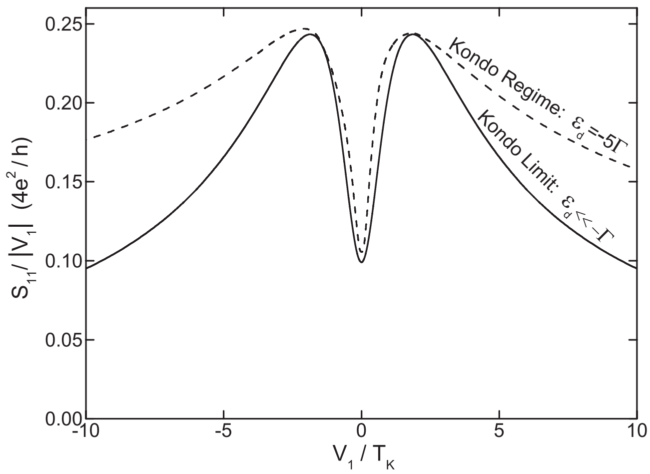

Figure 4. The voltage-scaled shot noise in lead 1 versus bias voltage V_1 for the beam splitter. Peaks around $|V_1| = 2T_K$ are obvious.

The same cross correlations were also presented in [9] with the parameter $\varepsilon_d = -6\Gamma$; however, the results are quite different from ours both in the lineshapes and scales. Figure 6(a) of [9] is so asymmetric that one can see nothing about the Kondo physics from it and the authors do not say which physical effect leads to such a asymmetry. Our results on S_{23} at $\varepsilon_d = -5\Gamma$ (figure 3(a)) also display asymmetric lineshapes, whereas we reveal that this asymmetry results from the broadening impurity level which can interfere with the Kondo resonance, as discussed in the above paragraph. There are two sources for the discrepancies between figures 2 and 3 of the present paper and the corresponding results of [9]. The first is due to the different positions of the level ε_d. The more important source is that our decoupling scheme differs from the SBMF theory. In the particle picture, the SBMF approximation is beyond the mean-field approximation and includes some parts of two-body correlations (in the quasiparticle picture, this approach describes noninteracting quasiparticles). To our knowledge, the correlation truncation scheme beyond the Lacroix approximation, used in the present paper, includes more two-body correlations than the SBMF theory, though a systematic investigation of the relation between the two approaches has not yet been made. The two-body correlations are expected to be important for current fluctuations in the strongly correlated regime of the Kondo effect.

Next, we consider the Y-shaped dot acting as a beam splitter. The leads 2 and 3 are identically biased ($V = 0$) such that a current is passed from the lead 1 into the leads 2 and 3, with no net current between them. In figure 4, we plot the voltage-scaled shot noise S_{11} for the beam splitter. It exhibits two peaks around $|V_1| = 2T_K$ both in the Kondo regime with $\varepsilon_d = -5\Gamma$ and in the Kondo limit. The heights of peaks ($\sim e^2/h$) and the lineshapes are consistent with that in a two-terminal QD [6, 25], except for a nonzero residual noise at $V_1 = 0$. A similar noise peak, also relating to the Kondo effect, was found in mesoscopic diffusive wires hosting magnetic impurities [26]. Here, we address that the nonzero S_{11}/V_1 at $V_1 = 0$ is an immediate consequence of a beam splitter with three leads. Actually an analytical expression for this quantity can be deduced from equation (9):

$$S_{110} \equiv \lim_{V_1 \to 0} \frac{S_{11}}{V_1} = \left\{ \frac{\Gamma_1(\Gamma_2 + \Gamma_3)}{\Gamma^2} \sin^2\theta - \left[\frac{2\Gamma_1(\Gamma_2 + \Gamma_3)}{\Gamma^2} \right]^2 \sin^4\theta \right\} \frac{16e^2}{h}. \tag{22}$$

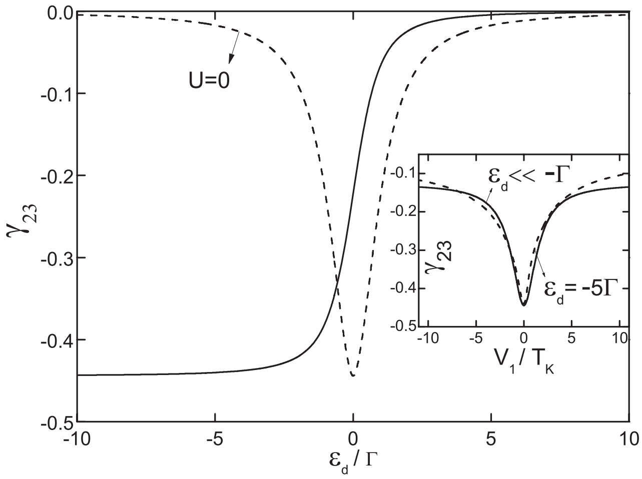

Figure 5. The cross correlator γ_{23} at $V_1 = 0$ as a function of ε_d. The dashed line corresponds to a noninteracting dot. Inset: γ_{23} versus V_1 in the Kondo regime and in the Kondo limit.

Using the present values of ε_d, we get $S_{110} \simeq 0$ for a two-terminal QD with identical couplings ($\Gamma_3 = 0$, $\Gamma_1 = \Gamma_2 = \Gamma/2$) and $S_{110} \simeq 0.1$ for our beam splitter.

Similarly, the Fano factors γ_{11} and γ_{23}, defined as $\gamma_{\alpha\beta} \equiv S_{\alpha\beta}/(2e\sqrt{|I_\alpha I_\beta|})$, at $V_1 = 0$ are readily found as

$$\gamma_{110} \equiv \lim_{V_1 \to 0} \gamma_{11} = 1 - \frac{4\Gamma_1(\Gamma_2 + \Gamma_3)}{\Gamma^2} \sin^2\theta \tag{23}$$

$$\gamma_{230} \equiv \lim_{V_1 \to 0} \gamma_{23} = -\frac{4\Gamma_1}{\Gamma^2}\sqrt{\Gamma_2\Gamma_3}\sin^2\theta. \tag{24}$$

Because γ_{110} and γ_{230} have similar dependence on the impurity level ε_d, we only plot the cross correlator in figure 5. γ_{23} at $V_1 = 0$ is a steplike function of ε_d, whose minimum value $-4/9$ is reached in the Kondo limit and maximum 0 in the empty-orbital limit. For comparison, the corresponding γ_{23} for a noninteracting dot is also presented, which is a Lorentzian function of ε_d. At $\varepsilon_d = 0$, the cross correlator γ_{23} for the noninteracting dot is exactly twice that for the dot in the strong correlation limit ($U \to \infty$). This is because it allows double occupation of electrons for $U = 0$, and single occupation for $U \to \infty$. When ε_d is deep below the Fermi level, the cross correlator in the interacting case decreases to $-4/9$ due to the dominating Kondo effect, while in the noninteracting case γ_{23} increases to zero because the transmission is very small. The dependence of γ_{23} on V_1 is only available by numerical calculations; see the inset of figure 5. We see that γ_{23} is minimal at $V_1 = 0$ and saturates at large voltages. The valley around $V_1 = 0$ is insensitive to the broadening effect and is scaled by the Kondo temperature T_K, though T_K has different values depending on the bare level ε_d. Our results for γ_{23} are in good agreement with the SBMF results [9]. The agreement indicates that the Fano factor γ_{23} is mostly determined by the mean-field effect and its changes are negligible, though S_{23} changes significantly, when more two-body correlations are taken into account.

4. Summary

We have studied cross correlations and shot noise in a three-terminal QD with the Kondo effect dominating. Both of them show a nontrivial dependence on the voltage applied to one lead when the Kondo resonance is split by the bias between other two leads. The cross correlations are more sensitive than the shot noise to the broadening effect of the impurity level and are always negative, reflecting the fermionic nature of the quasiparticles. When the QD acts as a beam splitter, the voltage-scaled shot noise, with a distinct residual noise at $V_1 = 0$, exhibits peaks around the Kondo temperature and the cross correlator shows a valley at $V_1 = 0$. Both quantities as functions of the scaled voltage V_1/T_K show a scaling behaviour, which can give a good estimate of the Kondo temperature. We have also compared our results with the corresponding SBMF results and the source of discrepancy has been discussed. Experimental tests of our results are possible, and we hope that the present work will indeed motivate experimental effort on the current correlations in multiprobe Kondo impurities.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant Nos 10375039 and 90503008, the Doctoral Education Fund of the Education Ministry of China, and the Research Fund of the Nuclear Theory Center of HIRFL of China. One of the authors, T F Fang, would like to thank X Min for helpful suggestions and kind encouragement.

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